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IDENTIFICATION OF BOUNDARY THERMAL PERTURBATIONS USING
SPECTRAL FUNCTIONS

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A method is proposed for solving the external inverse heat-conduction problem in a parametric formulation.

The heat-transfer boundary conditions at the surface may be determined from available information on the temperature inside an object, which forms a topic of the external inverse heat-conduction problem (IHP), by various methods [1-4], the application of which depends on the formulation of the IHP, the required accuracy of solution, and the presence of corresponding computational resources.

If the IHP is regarded as a control problem, in which the role of the control object is played by its model, the boundary conditions are taken as the input quantities, and the temperatures at the observation points as the output quantities, it is possible to speak of a correlation between the distributed input and output quantities, which may be expressed in the form of transfer functions or influence functions (the latter term, in our view, more closely corresponds to the physical meaning of this correlation).

In a particular case, determining the transfer functions of objects with distributed parameters (DP objects) consists in solving the heat-conduction equation for a single input perturbation at one of the boundary points, with zero perturbations at the other points of the surface [5]. The distributed transfer function from a single source at the given boundary point to a finite set of internal points of the given object is obtained here.

If, for each point boundary perturbation with amplitude f_i in the grid model of the object, the distributed function $W_i(x, y, z)$ is determined, where $i = 1, 2, \dots, N$, the W_i functions may be used to write the relation between its temperature and all the input boundary perturbations for all its internal points, under the condition that the DP object is linear

$$T(x, y, z) = \sum_{i=1}^N f_i W_i(x, y, z).$$

In this case, solving the IHP reduces to determining the amplitudes f_i . Unique determination of the function $f(x, y, z)$ entails having information on the temperatures at N internal points of the DP object and solving a system of N linear algebraic equations

$$T_j^* = \sum_{i=1}^N f_i W_{ij}; \quad j = 1, 2, \dots, N,$$

where T_j^* are the temperatures at the observation points of the DP object.

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Usually, in solving practical problems, limited information on the temperature values at the observation points is available to the investigator, and the input perturbations are determined in the form of parameters of the piecewise-constant approximation, which is the least precise of all those employed.

To increase the accuracy of IHP solution, the number of observation points must be increased, which leads to complication and increased expense of the experiment, to the need to store a large volume of numerical information, and to complexity associated with its treatment and analysis, to say nothing of the fact that increasing the volume of information and improving its quality are by no means always possible.

With the aim of eliminating these deficiencies, the following approach is proposed. If, for example, the distribution function of the input perturbations of the i -th section of the boundary is approximated by an n -th-order polynomial

$$f_i(\xi) = \sum_{j=0}^{n_i} a_{ij} \xi^j, \quad i = 1, 2, \dots, m; \quad j = 0, 1, \dots, n, \quad (1)$$

and the coefficients of this polynomial a_{ij} are taken as the parameters of the boundary perturbations, the temperature at any internal point of the object may be defined as

$$T = \sum_{ij} a_{ij} W_{ij},$$

and the function W_{ij} is the spectral influence function, since it characterizes the reaction of the object not to the total boundary perturbation but only to its spectral component. The elements of the sequence $\{\xi^k\}$, $k = 0, \dots, n$, are chosen as the spectral components of the boundary perturbations in Eq. (1).

The spectral functions of the boundary perturbations are determined as follows. On all boundary sections, except the i -th, zero perturbations are specified (for the case of boundary conditions of the first kind). Applying unit distributed perturbation to the i -th section of the boundary, for which the spectral influence functions are being determined, the Laplace equation is solved, and as a result the spectral influence function $W_{i0}(x, y)$ is determined. In determining $W_{i1}(x, y)$, it may be assumed that the distributed input perturbation is known in the form of the function $f_{i1} = \xi$; in determining $W_{i2}(x, y)$, that it is known in the form of the function $f_{i2} = \xi^2$; and so on. An analogous procedure is performed for the other sections of the boundary. After determining all the spectral influence functions, the relation between the output temperature potential and the input boundary perturbations may be written in parameterized form for any internal point of the object, as follows

$$T(x, y) = \sum_{i=1}^m \sum_{j=0}^{n_i} a_{ij} W_{ij}(x, y). \quad (2)$$

Equation (2) may be used to solve problems in both the direct and inverse formulation. In the second case, the input parameters a_{ij} must be determined; they are found from the solution of the system of linear algebraic equations

$$T(x_1, y_1) = \sum_{i=1}^m \sum_{j=0}^{n_i} a_{ij} W_{ij}(x_1, y_1), \quad (3)$$

$$T(x_{mn}, y_{mn}) = \sum_{i=1}^m \sum_{j=0}^{n_i} a_{ij} W_{ij}(x_{mn}, y_{mn}).$$

If the number of observation points N exceeds the number of initial parameters, the point least-squares method must be used for Eq. (2). According to this method, the measure of the deviation

$$I = \sum_{s=1}^N \left[\sum_{i=1}^m \sum_{j=0}^{n_i} a_{ij} W_{ij}(x_s, y_s) - T(x_s, y_s) \right]^2$$

is a function of the coefficients a_{ij} , which must be chosen so that its value is a minimum. Determining the partial derivatives of I with respect to each parameter and equating them to zero, a system of mn equations is obtained for determining mn unknowns

$$\begin{aligned}
 & a_{11} \sum_{s=1}^N W_{11}^2(x_s, y_s) + a_{12} \sum_{s=1}^N W_{11}(x_s, y_s) W_{12}(x_s, y_s) + \dots \\
 & + a_{mn} \sum_{s=1}^N W_{11}(x_s, y_s) W_{mn}(x_s, y_s) = \sum_{s=1}^N T(x_s, y_s) W_{11}(x_s, y_s), \\
 & a_{11} \sum_{s=1}^N W_{11}(x_s, y_s) W_{12}(x_s, y_s) + a_{12} \sum_{s=1}^N W_{12}^2(x_s, y_s) + \dots \\
 & + a_{mn} \sum_{s=1}^N W_{12}(x_s, y_s) W_{mn}(x_s, y_s) = \sum_{s=1}^N T(x_s, y_s) W_{12}(x_s, y_s), \\
 & \dots \\
 & a_{11} \sum_{s=1}^N W_{11}(x_s, y_s) W_{mn}(x_s, y_s) + a_{12} \sum_{s=1}^N W_{mn}(x_s, y_s) W_{12}(x_s, y_s) + \dots \\
 & + a_{mn} \sum_{s=1}^N W_{mn}^2(x_s, y_s) = \sum_{s=1}^N T(x_s, y_s) W_{mn}(x_s, y_s).
 \end{aligned} \tag{4}$$

Thus, the parameters of the piecewise-polynomial approximation of the boundary perturbations are determined as a result of solving Eq. (3) or (4).

Now consider the determination of the spectral influence functions of boundary perturbations in the nonsteady case, approximating the heat-conduction equation using an implicit finite-difference scheme

$$\Delta T^{(k)} - \frac{1}{\Delta Fo} T^{(k)} = - \frac{1}{\Delta Fo} T^{(k-1)}, \quad T^{(k)}|_{r_i} = f_i^{(k)}, \tag{5}$$

where ΔFo is the dimensionless time step; k is the number of the time step.

The output signal for the k-th instant of time is written in the form of the two components

$$T^{(k)}(x, y) = \sum_{i=1}^m \sum_{j=0}^{n_i} a_{ij}^{(k)} W_{ij}^{(k)}(x, y) + V^{(k)}(x, y). \tag{6}$$

The first component $\sum_{i=1}^m \sum_{j=0}^{n_i} a_{ij}^{(k)} W_{ij}^{(k)}(x, y)$ characterizes the reaction of the object to the boundary perturbation, while the second $V^{(k)}(x, y)$ characterizes the reaction to the initial perturbations — the temperature at the (k - 1)-th instant.

The spectral influence function $W_{ij}^{(k)}(x, y)$ is found as a result of solving Eq. (5) for the k-th time step, with zero initial and boundary conditions and a specified j-th spectral component of the boundary perturbation of the i-th section of boundary. The function $V^{(k)}(x, y)$ is determined from the solution of Eq. (5) with zero boundary perturbations and known initial perturbations.

For constant ΔFo and a linear formulation of the problem, the spectral influence functions are unchanged from step to step, since they depend on the geometry of the object, the thermophysical characteristics, and the magnitude of the time step.

In solving nonlinear problems, the equation

$$C_V(T) \frac{\partial T}{\partial \tau} = \nabla[\lambda(T) \nabla T]$$

is transformed using the Kirchhoff substitution

$$\Theta = \int_0^T \lambda(T) dT$$

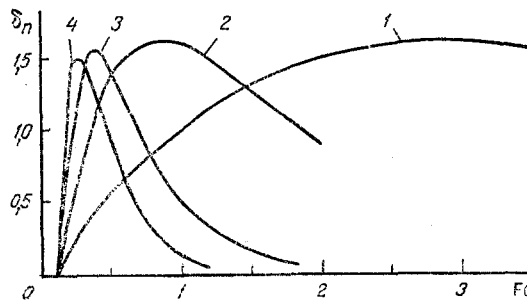


Fig. 1. Mean square deviations of identified surface temperatures from their specified (accurate) values for various cooling rates: 1) $\gamma = 0.25$; 2) 1; 3) 3; 4) 5. δ_n , %.

to an equation of the form

$$\Delta\theta = \frac{1}{a(\theta)} \frac{\partial\theta}{\partial\tau},$$

which, after applying the implicit finite-difference scheme, coincides with Eq. (5), except for the notation, and the remainder of the solution within the limits of the time step, with boundary conditions of the first and second kind, is actually no different from the solution of the problem in the linear formulation.

The procedure for finding W_{ij} is no different from that described above, but an iterative process with respect to the refinement of $W_{ij}(x, y)$ is applied at each time step, while the expression for determining θ is analogous to Eq. (6), since $a(\theta)$ is unchanged within the limits of the iteration.

The approach here proposed — determining the spectral influence functions of the boundary perturbations — is such that, while gathering information on the distributed boundary perturbations, the accuracy of their approximation is simultaneously increased (with the same volume of the initial information). In addition, decreasing the number of parameters to be determined improves the conditionality of the problem.

The spectral influence functions may be found both by numerical methods (for example, in hybrid systems [6]) and at an analytical level using the regional-structure method [7].

The division of the boundary of the given region into sections is determined, on the one hand, by the specific geometry of the object and the position of the observation points and, on the other, by the need to decrease the order of the approximating polynomial (within the limits of the section, a polynomial of lower order may be used than for the whole surface). The need arises because the approximation of the dependences to be identified by high-order polynomials may lead to instability of the solutions obtained [1, 4].

The efficiency of the above-described method is illustrated by the solution of a model nonsteady problem of determining the boundary thermal perturbations for a rectangular prism (with side ratio 1:2) using the finite-difference method. At the boundaries of the given region, temperatures are specified: $T(0, y, Fo) = (0.4 + 3.6y)\exp(-\gamma Fo)$, $T(x, 1, Fo) = 4(1 - x)^2\exp(-\gamma Fo)$, $T(2, y, Fo) = (0.4 + 3.6y)\exp(-\gamma Fo)$, $T(x, 0, Fo) = 0.4\exp(-\gamma Fo)$; the initial conditions are taken in the form of the solution of the same problem in a steady formulation with the same boundary problems. The results of the thermophysical experiment are simulated by the temperature values obtained at the points $P_1(0.1, 0.2)$; $P_2(0.1, 0.5)$; $P_3(0.1, 0.8)$; $P_4(0.2, 0.9)$; $P_5(1.0, 0.9)$; $P_6(1.8, 0.9)$; $P_7(1.9, 0.8)$; $P_8(1.9, 0.5)$; $P_9(1.9, 0.2)$; $P_{10}(1.8, 0.1)$; $P_{11}(1.0, 0.1)$; $P_{12}(0.2, 0.1)$ in solving the direct problem.

Analytical functions of the surface temperature in the inverse problem are determined for each of the four sections at the k -th instant of time, in the form

$$T^{(k)}|_{r_i} = \sum_{j=0}^2 a_{ij}^{(k)} y^j, \quad i = 1, 3; \quad T^{(k)}|_{r_i} = \sum_{j=0}^2 a_{ij}^{(k)} x^j, \quad i = 2, 4. \quad (7)$$

The additional conditions are the continuity conditions of temperature continuity at the corners. The variation in mean square deviations

TABLE 1. Parameters of the Boundary Perturbations for Different Times

Fo	a_{10}	a_{11}	a_{12}	a_{20}	a_{21}	a_{22}	a_{30}	a_{31}	a_{32}
0,1	0,361	3,257	0,000	3,619	-7,239	3,619	0,361	0,000	0,000
	0,361	3,257	0,000	3,619	-7,239	3,619	0,361	0,000	0,000
0,3	0,296	2,666	0,000	2,963	-5,926	2,963	0,296	0,000	0,000
	0,307	2,760	-0,046	3,021	-6,000	3,000	0,307	0,021	-0,011
0,5	0,242	2,183	0,000	2,426	-4,852	2,426	0,242	0,000	0,000
	0,259	2,339	-0,084	2,515	-4,954	2,477	0,259	0,042	-0,021
0,7	0,199	1,787	0,000	1,986	-3,972	1,986	0,199	0,000	0,000
	0,198	1,812	-0,111	1,899	-3,702	1,851	0,199	0,059	-0,029
0,9	0,163	1,463	0,000	1,626	-3,252	1,626	0,163	0,000	0,000
	0,165	1,520	-0,114	1,571	-3,043	1,521	0,165	0,062	-0,031
1,0	0,147	1,324	0,000	1,471	-2,943	1,471	0,147	0,000	0,000
	0,150	1,390	-0,114	1,428	-2,758	1,379	0,151	0,062	-0,031

$$\delta_n = \sqrt{\frac{1}{n} \sum_{s=1}^n [T^{(k)}(P_s)|_r - T_S^{(k)}(P_s)]^2}$$

of the identified surface temperatures from their specified (accurate) values over time is shown in Fig. 1. The approximation steps $\Delta x = \Delta y = \Delta Fo$.

The values of the coefficients $a_{ij}(k)$ are shown in Table 1 (upper row: accurate values; lower row: derived values) for functions characterizing the surface temperature of a rectangular prism for various Fo. Here $a_{10}(k) = a_{30}(k)$; $a_{11}(k) = a_{31}(k)$; $a_{12}(k) = a_{32}(k)$.

The approach here proposed is especially effective in solving multiparameter IHP, since the identified dependences are obtained in the form of analytical functions of spatial coordinates, which offers great possibilities for the express analysis of the thermal state of objects and the solution of control problems for thermal processes.

NOTATION

T, temperature; x, y, z, spatial coordinates; Fo, dimensionless time; ΔFo , step with respect to the Fourier number; ξ , coordinate along the boundary contour; a_{ij} , parameters of function of the boundary perturbations; W_{ij} , spectral influence functions of boundary perturbations; V, influence function from initial perturbations; C_v , volume specific heat; λ , thermal conductivity; τ , time; γ , cooling (heating) rate; δ_n , mean square deviation; s, number of boundary point.

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